

THEORETICAL NUCLEAR AND LASER PHYSICS



A MODEL STUDY OF THE INTERACTION OF AN ELECTRON WITH A GRASER BEAM

DISSERTATION

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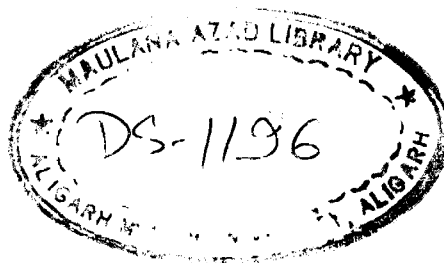
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GAGAN GUPTA



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TO
MY
PARENTS

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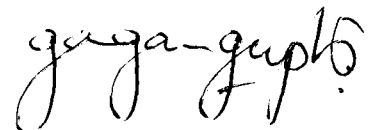
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A handwritten signature in cursive script, reading 'gagan-gupta'.

(GAGAN GUPTA)

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CONTENTS

Acknowledgements	i
Contents	iii
Chapter 1		
Introduction	1
Chapter 2		
A Brief Summary on X-Ray Laser Research	6
Chapter 3		
Graser Proposals Based on Nuclear Transition		8
3.1 Essential Requirements	8
3.1.1 Inversion Density and Cross-Section for Stimulated Emission		
3.1.2 Bandwidths		
3.2 Transition Possibilities	11
3.3 Systematic Steps for the Search of Appropriate Graser Candidates	14
3.4 Isomer Separation	16
3.5 Proposals for Pumping Radiations	18
3.5.1 Two-Step Pumping		
3.5.2 Optical Pumping		
3.5.3 Miscellaneous Pumping Techniques		
3.6 Nuclear Superradiance	27
3.7 Miscellaneous Nuclear Proposals	29
Chapter 4		
Graser Proposals Based on Electron and Positron Beams		32

4.1 Interaction between an Electron Beam and Two Interfering Laser Beams	32
4.2 Electron-Positron Graser	35
4.2.1 Induced Annihilation of Para-Positronium Atoms		
4.2.2 Relativistic Electron-Positron Graser		
4.3 Other Ideas	40
 Chapter 5		
Proposed Applications of Grasers	41
 Chapter 6		
Electrodynamics in a Graser Field	44
6.1 Background to the Problem	44
6.1.1 Charge Particle Dynamics in the Electromagnetic Field of a Plane Wave		
6.1.2 Charge Particles in a Laser Field		
6.2 Interaction of an Electron with a Graser Field		50
6.2.1 General Description		
6.2.2 Calculation and Results		
6.2.3 Conclusion		
 References	72

CHAPTER - 1

INTRODUCTION

The rapid development of laser technology since the first Ruby laser of Maiman (1960) has made it almost impossible to provide a comprehensive list of laser applications at any point. Lasers consisting of different materials can be as small as a pinhead (coupled with integrated optics) and as large as a football field (the free electron laser). The present state-of-the-art lasers can emit light from X-rays to far infrared radiations.

According to Einstein's formula, the probabilities of spontaneous (A) and stimulated (B) emissions at wavelength (λ) are connected through

$$A/B = 8\pi h/\lambda^3$$

This shows that for shorter wavelengths spontaneous emission becomes a strong competitor to stimulated emission. It has [1] been shown that, in principle, amplification by stimulated emission can be obtained upto all wavelengths, larger than 1 picometre.

After developing the excimer and free electron lasers which are capable of efficient generation of high power pulses of radiation in the ultraviolet and vacuum ultraviolet regions, the eyes are now set towards even shorter wavelength

lasers, viz., X-ray lasers and gamma ray lasers (grasers). At present Japanese scientists have found indications for [2,3] 0.78 nanometre X-ray lasing.

In Chapter 2 we will first briefly summarise the steps made for the development of soft X-ray lasers together with their proposed applications. It also includes the ideas for the further development of shorter coherent X-ray emission.

Although the possibility for coherent gamma ray emission was first introduced in 1961 [4-6], yet despite of several proposals no one has succeeded in developing a graser. Chapters 3 and 4 review all major proposals given by different authors for developing a graser. The latest major review articles [1,7] on this subject were written in 1981 and 1982. In this dissertation we have up-dated our knowledge to autumn 1987.

Chapter 3 reviews the proposals for developing a graser based upon the nuclear transitions. Sections 3.1 to 3.6 discuss the major proposals based on nuclear transitions. The last section of this chapter, 3.7, briefly describes some other under-developed ideas for developing a graser based on the nuclear transitions. Section 3.1 discusses the essential requirements for the lasing conditions in gamma-ray region, viz., the inversion density and cross-section for stimulated emission and bandwidths. Section 3.2 deals with the various transition possibilities leading to lasing

radiation in gamma-ray region. It finally reduces into a search of suitable isomeric nuclei which may exploit the Mossbauer and Borrmann effects. In section 3.3 our intention is to discuss the progress made by the various laboratories for the search of proper graser candidates. It is expected that for developing a graser one has to first isolate sufficiently pure isomeric nuclei of the appropriate graser candidate. Section 3.4 deals with the nuclear isomer separation problems and gives an up-dated knowledge of experiments done by different groups. Section 3.5 deals with the pumping processes for the upconversion of isomeric nuclei to the upper laser level. References 1, 7, 46 and 63 describe a long list of proposals for the pumping radiations together with their merits and demerits. But here we wish to deal with the prominent proposals only. At the present state-of-the-art the two-step pumping schemes seem to be the most fruitful. The subsection 3.5.1 describes the two-step pumping proposal in brief while the subsection 3.5.2 deals with the optical pumping mechanisms as a part of the two-step pumping schemes. The last part of the section 3.5 introduces some new alternative mechanisms for the two-step pumping. After transferring the population to the upper laser level it is justified to discuss the lasing process. Nuclear superradiance is supposed to be a viable lasing process in the case of grasers, about which the section 3.6 deals in

detail.

Besides the proposals based on the nuclear transitions discussed in Chapter 3, Chapter 4 describes the proposals based either on the electron beams or on the electron and positron beams. Section 4.1 deals with a proposal which is analogous to the development of free electron lasers. Section 4.2 deals with electron-positron grasers. The last part of Chapter 4 gives a list of some under-developed ideas for the development of non-nuclear grasers.

It is a human propensity to speculate the possible applications of an under-developed instrument. Similarly, in the case of grasers, people have started to foresee the possible applications of grasers. Chapter 5 gives a comprehensive collection of applications in science and industry.

Chapter 6 deals with the problem of interaction of an electron with the graser beam. Although it is a model calculation of the problem, it yields very interesting results. Subsection 6.1.1 briefly reviews the charge particle dynamics, classically, in the electromagnetic field of a plane wave. Under the dipole approximation the radiation field generated by the laser system may be treated as the plane wave if the wavelength of the radiation field is larger than the charge particle mean free-path. Subsection 6.1.2 quantum mechanically, describes the behaviour of an

atomic electron inside the laser field. It also describes the experimental work for this behaviour.

In section 6.2 we have presented a model calculation of our own for the interaction of an electron with a graser beam. Our results indicate interesting physics in the form of either reflection or transmission or trapping of the electron for large times. However, it is a model calculation and the complete solution to the problem requires the introduction of some other factors, as discussed in the conclusion part of section 6.2, these results are not expected to be washed out.

CHAPTER - 2

A BRIEF SUMMARY ON X-RAY LASER RESEARCH

At University of Tokyo scientists have found indications of X-ray laser emission at 0.78 nanometre wavelength [2,3] while the results in Rutherford Appleton Laboratory [16] and in Lawrence Livermore National Laboratory [17] show emission at 8.0 and 10.5 nanometres respectively. All of these three experiments involve the use of powerful optical lasers by focussing their short pulses on a material raising its temperature to about 10^7 °K and pressure to about 10^8 atmospheres. This converts the material into a hot plasma. The population inversion is achieved in the highly ionized atoms by means of collisional pumping mechanisms. Using the Nova laser facility at Lawrence Livermore National Laboratory scientists have achieved lasing at 10.5 nanometres wavelength and output powers to be about 1 - 10 Mega Watts in a pulse width of 175 picoseconds [17].

It is expected that this technology may not be fruitful for producing hard X-ray lasers, which may require an entirely new excitation technique. X-ray laser action at 1.2 nanometres has also been reported [18,19]. This emission was pumped through thermonuclear explosion. Moreover, the concept of free electron lasers in X-ray region with the

optical wigglers suggests coherent X-ray emission at 0.5
[20]
nanometre .

It has been suggested that future developments including the use of particle accelerators for excitation of muonic atoms, charge-annihilation and other novel techniques may lead to population inversions required for the
[21]
development of even shorter wavelength lasers .

Since the penetrating power of X-rays is not very high, therefore they can be used for the study of living cells. A soft X-ray laser may be useful in high resolution microscopy through three dimensional holography. It may be an efficient tool for the study of physiochemical properties and the roughness of surfaces and also in molecular
[18,22]
physics . Further, the X-ray lasers operating between 2.3 - 4.4 nanometres (water window, the region in which the X-rays can pass through water but are absorbed by carbon based materials) may be used in the determination of
[23]
biological structures .

The use of X-ray lasers in defence can not be ignored. Very recently on December 23, 1987, U.S. defence scientists have reported investigations on Lithium like Aluminium X-ray
[11]
laser under the "Star-Wars" missile defence system .

CHAPTER - 3

GRASER PROPOSALS BASED ON NUCLEAR TRANSITIONS

In this section we classify approaches and discuss the problems confronting the developers of grasers that might use nuclear transitions in the 5 - 100 keV region (10^{-9} centimetre $< \lambda < 2.0 \times 10^{-8}$ centimetre, the ultra short wavelength band).

[7]

3.1 Essential Requirements

3.1.1 Inversion Density and Cross-Section for Stimulated Emission

For amplification of radiation by stimulated emission from an upper state to a lower state the photons must be added by transitions between the two levels into a limited number of modes of the radiation field more rapidly than they are removed. Thus for lasing action the inversion density n^* ($n^* = n_2 - (g_2/g_1)n_1$, where n_2 , g_2 and n_1 , g_1 are the population densities and degeneracy factors for the upper and lower levels respectively) must exceed the ratio

$$n^* > (\sigma_a / \sigma_s) \cdot n \quad \dots(1).$$

Here n is the total atom density, σ_a is the average cross-section of the medium for absorbing or scattering a photon. In normal matter that is not in the form of plasma, it can be expressed as

$$\sigma_a = CZ^{4.5} \lambda^{-3} \dots (2)$$

where the constant C depends upon the material. Z is the atomic number and λ is the wavelength of electromagnetic radiation.

$$\sigma_s = (\lambda^2 / 2\pi) \cdot (\Gamma_r / \Gamma) \dots (3)$$

σ_s is the cross-section for stimulated emission. Γ_r and Γ are the radiative and total linewidths of the energy level, respectively. Thus the threshold inversion density $n^* = (\sigma_a / \sigma_s) \cdot n$ is proportional to the wavelength. Hence for gain at shorter wavelengths in normal matter, a smaller excess of excited-state population density is needed. But in gamma-ray region, the problem whether that excitation can be produced without converting the medium to a plasma in which the resonant absorption of photons is also important, remains to be resolved.

3.1.2 Bandwidths

In equation (3) the ratio of radiative width to the total linewidth is called "line-broadening factor" and the other term of $(\lambda^2 / 2\pi)$ is known as "area factor". This area factor exceeds σ_a by 10^4 in ultra short wavelength region. Therefore, from equation (1), the gain equation, the line-broadening factor is crucial. This factor attains the maximum value of unity only when the transition is purely radiative and to a ground state.

In general the transition also involves non-radiative as well as the inhomogeneous processes. The total linewidth can be taken as the sum of the intrinsic widths of the two energy levels, to their finite lifetimes, and any additional broadening associated with random displacements of the transition energy by locally varying fields or temperature. Thus,

$$\Gamma = \Gamma_r + \Gamma_{\text{non-rad}} + \Gamma_{\text{inhomo}}$$

$$\Gamma = T_2^{-1} + T_1^{-1} + \Gamma_{\text{inhomo}}$$

where T_2 and T_1 are the lifetimes of the upper and lower energy levels, respectively. In the absence of inhomogeneous broadening the radiative width, Γ_r is given as

$$\Gamma_r = b T_2^{-1} (1 + \alpha)^{-1}$$

where b is the fraction of all non-radiative decays from a given level that terminate on a second given level and α is the ratio of the probabilities of a non-radiative and radiative processes.

Thus in order to satisfy the gain condition, equation (1), the radiative width Γ_r must be an appreciable fraction of the total linewidth Γ . Inhomogeneous broadening should therefore be slight, and the non-radiative transitions, viz., Auger effect in atomic and internal conversion in nuclear transitions must not dominate the transitions, i.e., α must be small.

3.2 Transition Possibilities

We can see from equations (1), (2), and (3) that for ultra short wavelengths the threshold inversion density, n^* , needs a smaller (about 0.1%) excess of excited state population. Since the excited state lifetimes usually tend to decrease as transition energies increase, so the power demand on the pump presents a serious problem.

In ultra short wavelength region the atomic radiations originate in E2 transitions of single electrons. The lifetimes of excited atomic levels are in femtoseconds or less. [25] It has been suggested that if a plasma is pumped with fast neutrons with lifetimes of the order of picoseconds then it might accumulate inverted populations capable of lasing and for such short-lived states the inhomogeneous broadening becomes comparable with normal broadening. Thus for lasers, in order to fulfill the gain condition, it is doubtful that inversion can be accumulated in such short-lived atomic excited states.

Although nuclear transitions require equally high energy density for inversion, they do not necessarily demand high power density, since the lifetimes of radiatively decaying nuclear isomer states can be many orders of magnitude longer than those of atomic states.

Most important, the Mossbauer effect reduces the limitations imposed by the inhomogeneous broadening. In

order to achieve high cross-section for stimulation, the active isomer nuclei must be in a solid host at a temperature well below its melting point. Moreover, for radiations interacting with isomeric nuclei arranged in an ordered array at regular lattice situations within a solid single crystal, the Borrmann effect [34,47,50,53,62], in analogy to the Bragg-reflections, can enhance the radiations and hence leads to relax the inversion density requirements.

The lower states of transition in a polarized non-zero magnetic moment nuclei can remove the resonant absorption while the excited states of such nuclei can help in separating the excited and unexcited nuclei.

The power required to invert transitions having lifetimes in the Mossbauer range is high enough (although it is less than would be needed for pumping atomic transitions) to cause noise--temperature rise and lattice damage that can destroy the physical conditions required for the Mossbauer and Borrmann effects. Thus in order to develop a workable graser, it must be devised to either (1) nondestructively pump a Mossbauer transition, or (2) eliminate, or at least greatly reduce, all of the various line-broadening effects that prevent Mossbauer emission from long-lived transitions.

Most proposals for grasers have avoided a resonator and ensure the beam formation by making the active medium in needle-shaped form. However, it is, of course, acknowledged

that the coherence of the spontaneous radiation would not be as high, nor would the excitation be used as efficiently as with a resonator.

3.3 Systematic Steps for the Search of Appropriate Graser Candidates

Current plans for developing the graser are confronting various interdisciplinary problems. At present a major problem is to identify the proper material(s) from the different parts of the periodic table. A laser-grade database of nuclear properties requires highly precise experiments for the nuclear properties. At Los Alamos National Laboratory (LANL) [14,46], scientists have examined the appropriate experimental and theoretical data and have indicated the most fruitful mass regions. They have identified some pairs of the isomeric states (lifetimes > 5 seconds) with short-lived levels within a specified excitation energy window of width 1 - 5 keV, viz., Os, Ir, Pt, Au, Hg, etc. They have found that odd-odd nuclei in the rare-earth region would be a likely place to begin a search for appropriate graser candidates and on the basis of their nuclear model they find Re as a viable candidate.

[32,49]

The Rochester-Stanford joint research programme for the evaluation of graser candidates has developed an experiment using tandem Van de Graff accelerator with the study of high spin states in deformed nuclei and on recoil mass spectrometer (RMS). It is hoped that this joint

programme will take the results from LANL and evaluate the proper graser potentials.

At University of Texas, Dallas, a group working under the Strategic Defence Initiative project [40,59] has now identified 29 candidates out of the 1886 proposed candidates which are now planned to be examine on an X-ray flash lamp of nanosecond duration that can emit a total of 1 Joule per keV of linewidth. The ultimate success of these pump schemes involving flash X-rays will require investigation of the nuclear properties of those materials that are analogous to the kinetics of a conventional laser medium. This examination of selected candidates can also be performed in a little time by means of using either the laser plasmas or large electron beam machines but the economics involved in it do not permit it for 29 candidates. The experimental set up for the flash X-ray device is near to the threshold needed for the investigation.

3.4 Isomer Separation

From the discussion made in the previous section it is clear that for developing a graser based upon nuclear transitions one has to invert the ground state population to the isomeric level and to separate such levels from the rest of the other nuclear reaction products to get the isomeric enrichment [46]. In literature, there are some laser techniques for isomer separation, analogous to laser isotope separation. These are: (1) Resonance ionization--in which the ionization step is performed by laser beams. Dyer et al. [31,76] have demonstrated the isomerically selective photoionization of ^{197m}Hg nuclei. (2) Optical piston--in which the separation is done by exciting of the isomeric atoms from the rest of the mixture being kept in a capillary cell [77]. Dyer has suggested this technique as a model for atoms like Sodium [31]. (3) Atomic beam methods (a)--in which the isomeric atoms in a beam are first optically pumped into particular magnetic substates with a circularly polarized laser. They can then be filtered by means of a magnetic-moment analyzer [1]. (b)--Alkhazov et al. [78] have shown the resonance ionization of ^{142m}Eu by three laser beams. (4) Photochemistry--chemical separation of the excited atoms possessing the isomeric nuclei.

[7]
In radiochemical methods the enrichment of isomeric nuclei, being produced in nuclear reactions, is achieved

through Szilard-Chalmers process, in which recoil energy of the nucleus is greater than the chemical binding of the nucleus in a compound.

Despite of all these techniques, the optimum choice of any isomer enrichment process will depend on the properties of the actual graser material.

3.5 Proposals for Pumping Radiations

The use of nuclear transitions for generating stimulated emission in ultra short wavelength region has been proposed nearly 25 years ago. In this section we will briefly discuss the proposals given for pumping the population to a suitable Mossbauer energy level. In practice, the pumping radiation must be intense--to provide an adequate concentration of active isomer, efficient--so that a negligible part of pumping radiations be expended in heating and specific--so that the host may not be polluted.

The previous sections describe a viable pumping process for graser development through Mossbauer and Borrmann effects. But the time required in the whole process, i.e., to activate, separate, concentrate and crystallize the isomeric nuclei demands much longer transitions (about 10^{-5} second). Moreover, for long-lived nuclear transitions, the resonance cross-section is far less to meet the line-broadening requirements. On the other hand, the high intensity pumping may overheat or destroy the solid host which must support recoilless emission.

3.5.1 Two-Step Pumping

Despite of several proposals [15,63] no one has succeeded in beating this dilemma. At present, the two-step pumping scheme seems to be the best proposal, which we will

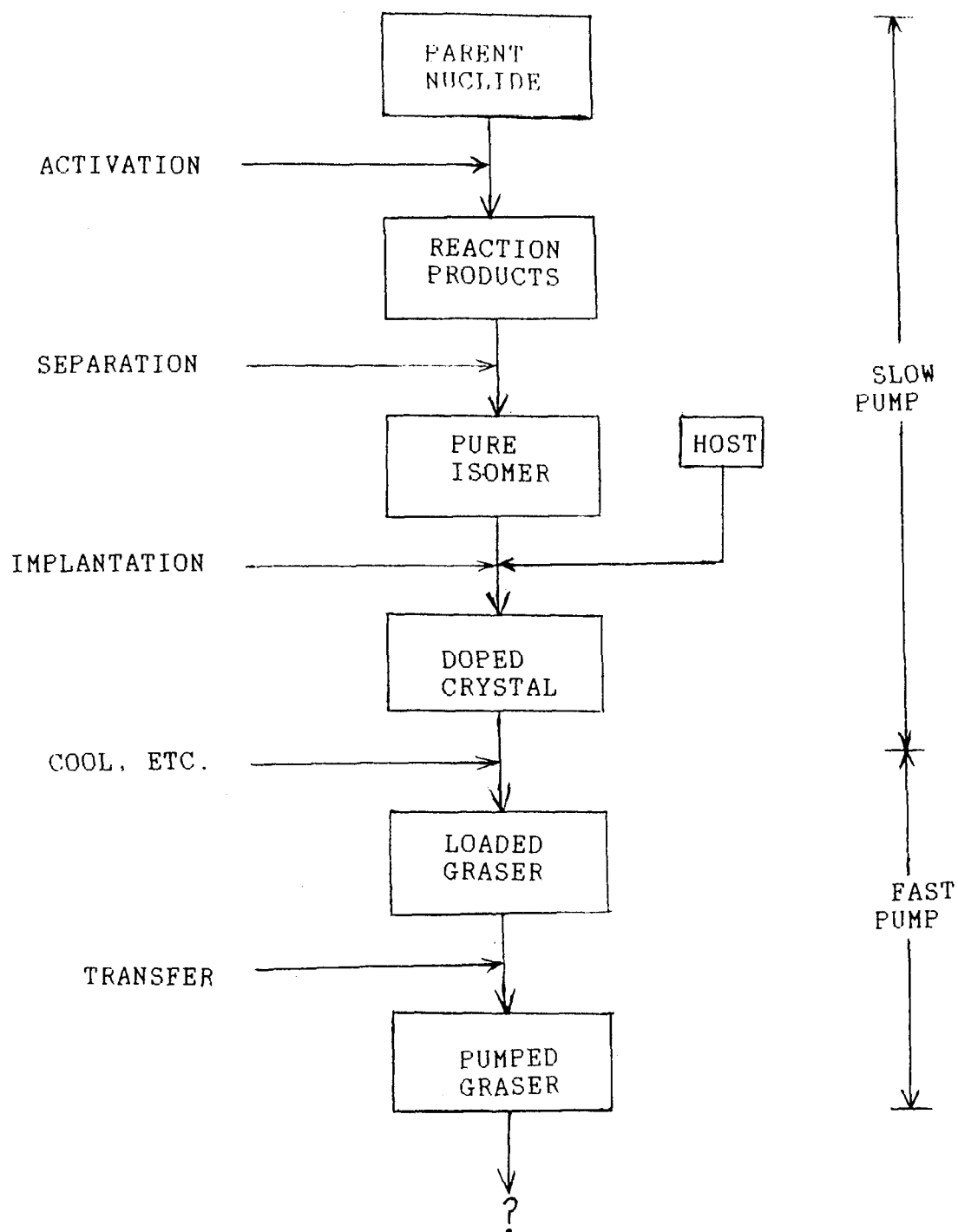


Fig. 1: Essential operations in the two-step pumping concept.

now discuss in detail (see Fig. 1). This scheme for reducing pumping power envisages embodying a "storage isomer" into a host and then transferring the population from the isomer to a Mossbauer level that can lase. It requires a short-lived and broad level very close (in energy) to the storage level.

In two-step pumping schemes the existence of the transfer level can not be resolved unless some special spectroscopic techniques are involved. Haight et al^[39] have suggested the possibility of exploiting broad-band radiations to excite such a level and then to confirm its existence. However, this method for searching such transfer levels would be useful only under some very restrictive conditions. If such a suitable graser candidate exists then the production and separation of the isomer nuclei from other nuclear products have to be tailored. At present, it can be done at least in some particular cases as discussed in section 3.4. The feasibility of this scheme also depends upon the rapid and efficient population transfer process. These transfer process proposals, viz., transfer by resonant radiation through optical pumping or through atomic excitation, will be discussed in detail in subsections 3.5.2 and 3.5.3, respectively.

3.5.2 Optical Pumping

Two-step upconversion processes for optically pumped nuclear reactions can be divided into coherent and incoherent categories.

The viability of these pump schemes depends upon the suitable nuclear material as well as on the efficient path of cascading from the intermediate short-lived level to the upper laser level.

The adjacent sketch (see Fig. 2) shows the schematic diagram for the energetically excited levels of a typical nucleus of interest to the development of a graser. The arrow (1) illustrates the incoherent pumping of the storage level through the absorption of X-rays (from laser plasma or an exploding wire) that are resonant with the energy separation between the storage level and the next higher level of proper symmetry. The arrow (2) represents coherent pumping through the non-resonant absorption of a photon from the radiation field in order to create a virtual or dressed state of excitation. Ultimately the graser output results from the upper laser level populated by a cascade.

Coherent Pumping

In this technique the nuclear isomeric state is excited to a virtual (or dressed) state with high intensity optical pumping (the anti-Stokes upconversion of conventional laser

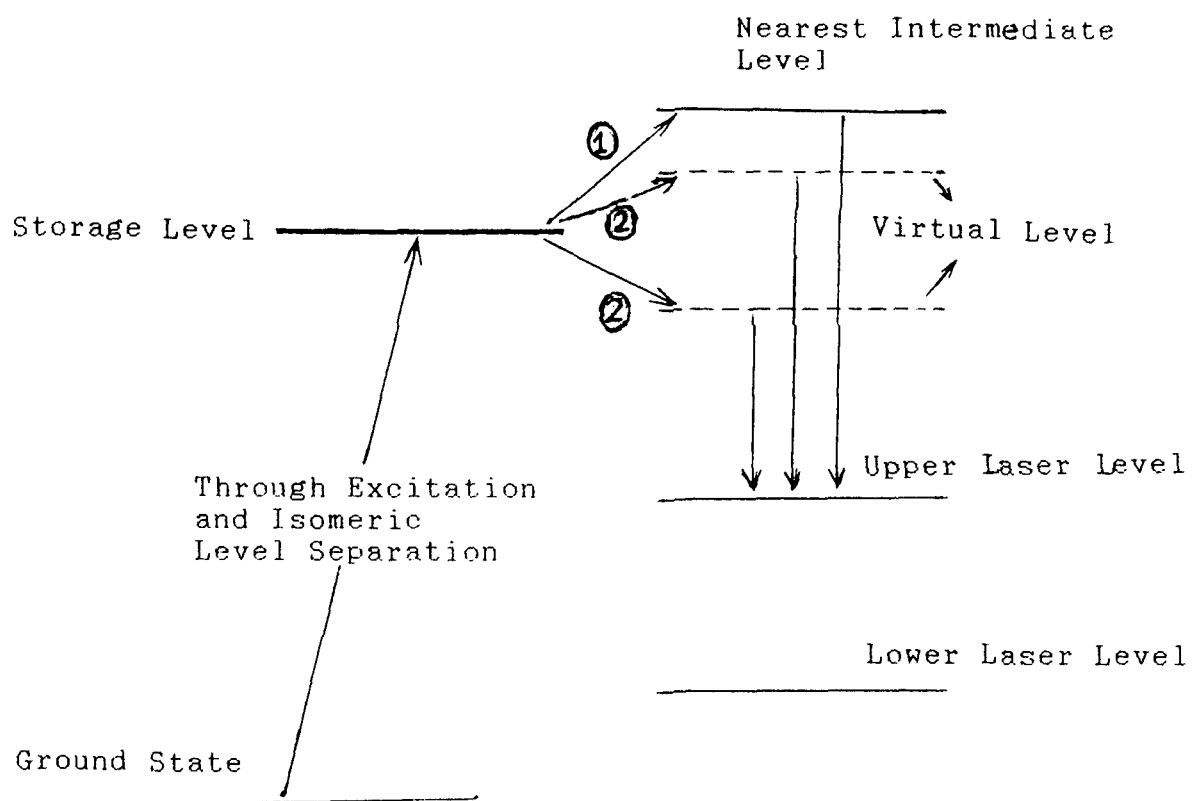


Fig. 2: Coherent and incoherent pumping schemes.

radiation, see Fig. 2). The populated virtual level decays to the laser level leading to the stimulated emission. The stimulated emission cross-section for the composite process, i.e., transferring the population from storage level to virtual level and then to decay of the virtual level to the upper laser level, can be given by the Breit-Wigner value as

$$\sigma = \lambda^2 / 2\pi$$

where λ is the wavelength of the stimulated emission. In order to diminish the direct transitions between the isomeric state to upper laser level, the natural width has to be preserved.

The possibility of the temperature rising in the pumped dressed layer can be avoided by diluting the active nuclei in a low Z host, e.g., ⁹Be. Quantitatively it has also been [64] computed ³ that for a critical intensity of 5.3×10^5 Watts per centimetre ² and a typical ⁶ 10 Hertz spurious broadening the threshold fluence should be 167 mili Joules per centimetre ². This proposal describes a model with a laser medium of length 1.0 centimetre and a filament of diameter 0.04 centimetre and the requirement that each pump photon makes ten trips of the laser medium. For this pumping threshold would be about 26 Joules in a pulse of duration 300 nanoseconds and the output obtained will be 66 Joules at 10 keV. Moreover, the laser threshold might be further reduced by manipulating the bulk ferromagnetic or ferroelectric

properties of the material in which the nuclei are
[55,66]
diluted.

Incoherent Pumping

In incoherent pumping of nuclear material, the resonant excitation of energetic states of nuclei is done by the X-radiation from laser plasmas or exploding wires. Theoretical estimates show that a transfer level decaying after 10 picoseconds through an E1 transition to the upper laser level with 10 keV will have 6.6×10^{-5} eV bandwidth, assuming the complete resonance between the pump and transfer level. So the complete fluence from the optical to the upper laser level can be built up for a time equal to its lifetime (1 - 10 nanoseconds).

For an X-ray laser plasma source (radius, R, of about 30 micrometres and length, L, of about 0.1 centimetre), the total population of upper laser level can be given as

$$N_u = (n \sigma_x N_0) / (2\pi RL)$$

where n is the number of X-ray photons produced in the source during the lifetime of upper laser level, N_0 is the concentration of transfer level and σ_x is the Breit-Wigner cross-section for the X-ray wavelength. This model further suggests that a relative fraction of about 10^{-5} eV of the line energy would be used in E1 transitions. For an X-ray energy around 10 keV, the threshold value is 10 Joules in the

X-ray line and results in a threshold fluence of 131 Joules per centimetre² for a 10 keV output transition. However, this threshold can further be reduced by manipulating the properties of the material embedding the nuclei.

3.5.3 Miscellaneous Pumping Techniques

An alternate to direct photoabsorption for pumping the nuclear transition from the storage isomeric level to the short-lived level is the possibility to induce transfer by exciting the atomic electrons with ultraviolet lasers at 10¹⁴ - 10¹⁷ Watts per centimetre² intensity [36,37,51] leading to [35,47,51,58] strongly mixed atomic-nuclear states. Since the electron system in the near field of the nucleus possesses large multipole moments and broad resonances, such mixed states can supply energy, angular momentum and parity to the storage isomer level either through resonant or through the collective electronic excitations [28,35,51,56,57]. The lowered multipole moments for the isomeric level decay bring a large change in the internal conversion coefficient which ultimately decreases the lifetime of the isomeric state. This approach may relax the pumping problems by direct field induced transfer [44,45] [61]

Anderson et al. have reported the nuclear fluorescence observations from the Mossbauer level, being pumped through broad absorption band radiations from the

isomeric state.

Reference 67 proposes an alternative technique in which absorption of X-ray photons, produced through inner-shell atomic transitions, might excite the nuclear electromagnetic transitions leading to the amplification of gamma radiation.

3.6 Nuclear Superradiance

The pumped upper laser level (Fig. 2 of section 3.5), assuming ideal conditions for material selection, isomer enrichment and a viable pump scheme required to develop a graser with a two-step pumping mechanism, will yield the amplified gamma radiations. The ordinary stimulated emission processes in gamma ray region require good mirrors for such short wavelengths which do not exist yet. Moreover, the large internal conversion coefficient in Mossbauer transitions rapidly depopulates the upper laser level and hence leads to poor inversion density. Therefore, on considering the line-broadening calculations, the ordinary laser amplification requires a long buildup time. For these reasons the preferred coherent gamma ray emission mechanism from the upper laser level is to be superradiance. The intensity and pulse width of a superradiant output pulse varies as N^2 and N^{-1} , respectively, where N is the population enrichment in the superradiant upper laser level. Such a state can be formed in a single crystal of low absorption coefficient for the outgoing radiations [62,80,81]. Feld and Baldwin [29,63,82] have given model superradiance calculations for ^{119}Sn (diluted in diamond) and for ^{133}Ba (in borazon). The output performances of their results are shown in Table 1.

Table - 1

Assumed and calculated parameters	Nuclide	
	¹¹⁹ Sn	¹³³ Ba
Length of SR sample (centimetre)	1.0	1.0
Diameter of SR sample (centimetre)	8.1×10^{-5}	1.13×10^{-4}
Active nuclei in upper laser level	7.8×10^{12}	5.0×10^{13}
Peak Power (Mega Watts)	4.90	35.9
Pulse width (nanosecond)	1.64	0.67
Power enhancement	5.0×10^9	6.74×10^9

SR sample: material sample containing superradiant nuclei

3.7 Miscellaneous Nuclear Proposals

The major nuclear proposals for constructing grasers have been presented in the preceding sections of this chapter. We now turn to some more ideas which have only been initiated so far.

According to Collins et al [27,64] the optical pumping for transferring the population from the storage level to the short-lived state (see Fig.2 of section 3.5) requires an energy fluence of about $131 \text{ Joules per centimetre}^2$ to be deposited in the graser material which produces a pressure of about $10^6 \text{ Newtons per centimetre}^2$ and temperature of about $10^4 \text{ }^\circ\text{K}$ provided the lifetime of short-lived level is about 10^{-6} second. Such a high temperature can abolish the Mossbauer conditions. An alternative proposal to beat this dilemma by means of electronic heat conduction under high pressure is given by Winterberg [43]. It suggests that the graser material be kept either in a static high press or in a high explosive pressurising geometry (see Fig. 3). Provided the outer medium is of good heat conductivity and at low temperature, the graser material can be made to lose sufficient heat within the pumping time.

[38] Hoy has suggested an experiment for searching for long-lived Mossbauer nuclear levels with lifetimes of about 1 second, which may be exploited in developing a graser. At present no such level is known.

Since the high intensity pumping of population from an

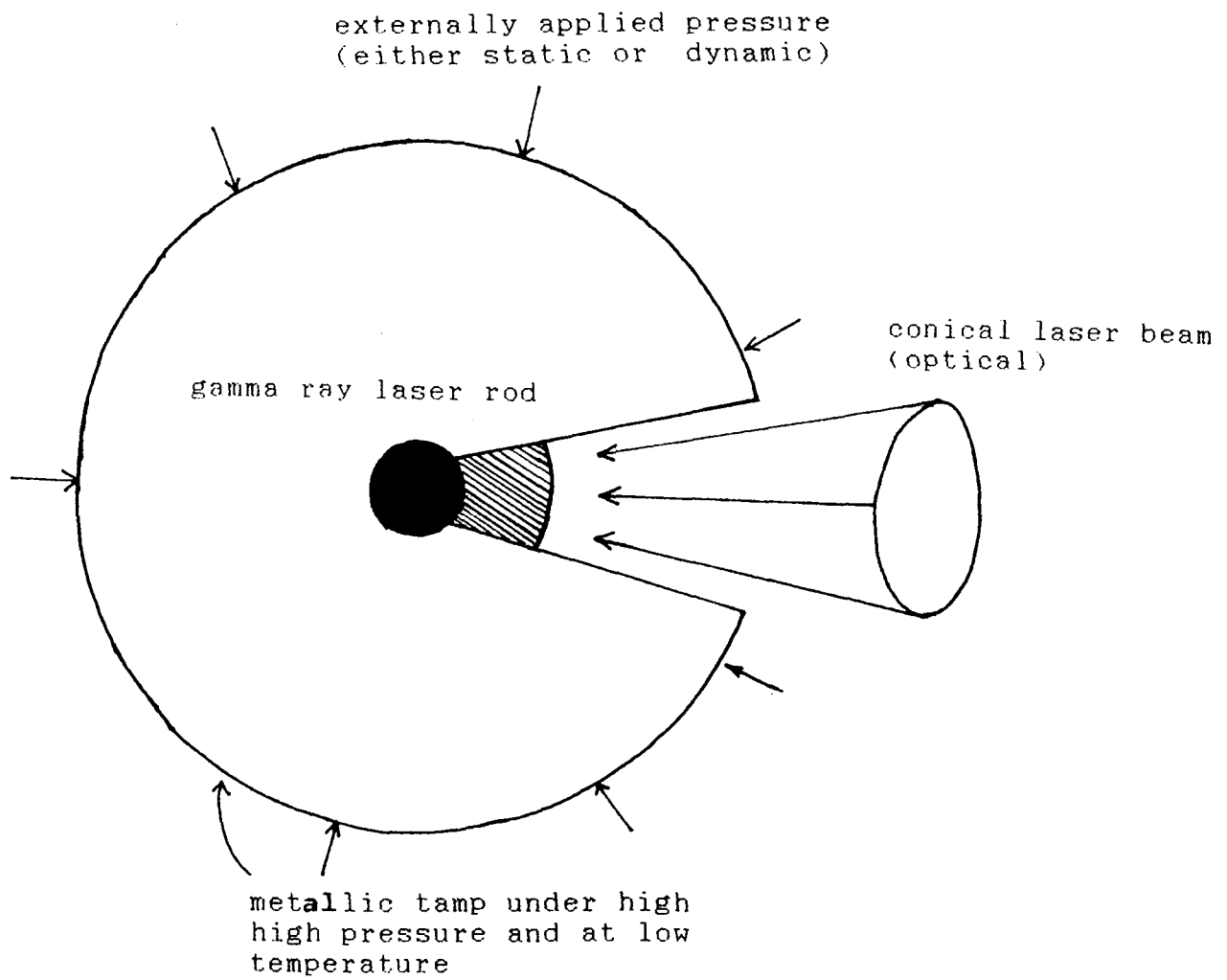


Fig. 3: Rapid heat-removal under high pressure.

isomeric level to Mossbauer level may destroy the Mossbauer and Borrmann conditions, Andreev et al.^[75] have proposed an alternative "Q-switching" multi-pumping process, in which the recrystallization of the Mossbauer level is to be done by optical laser annealing process.

In addition to the above proposals some other ideas based upon giant resonances, nuclear hole states and nuclear excimers have also been mentioned but no detailed work has been reported yet^[27]

CHAPTER - 4

GRASER PROPOSALS BASED ON ELECTRON AND POSITRON BEAMS

4.1 Interaction between an Electron Beam and two Interfering Laser Beams

[68,79]
This proposal, analogous to free electron lasers, reviews the possibility of amplifying the scattered radiation of two interfering laser beams of nearly same frequency and same intensity to produce coherent gamma rays. [68]
It has been shown that in a particular interaction geometry (see Fig. 1) the probability for producing the coherent gamma rays may be appreciable. In this geometry an electron beam crosses a periodic structure of interference fringes produced by two laser fields. An electron undergoes a scattering process with the photons and hence the photons are scattered in arbitrary directions. These photons may be backscattered along beam 1 or 2 through an inverse Compton scattering. If the electron energy is high enough to backscatter the photon along beam 2 (in the opposite direction) then the scattered photon gains energy from the relativistic electron beam. Under the particular angular conditions, the photons backscattered by each fringe may add in phase, so that the total probability for producing coherent gamma radiations is proportional to N^2 , where N is the total number of fringes that a photon crosses. If , the semiangle between the two interfering CO laser beams of

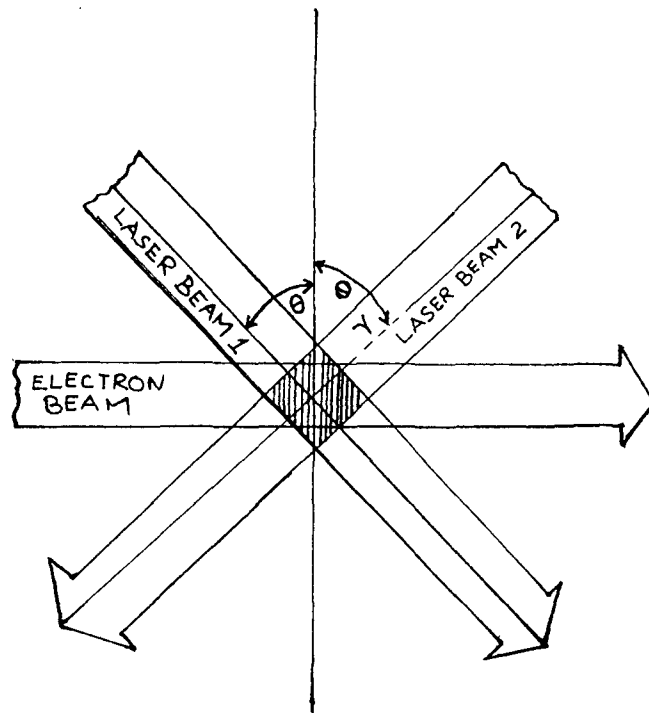


Fig. 1: Geometry for the production of gamma rays.

10.6 micrometres wavelength, is about 1° , then a relativistic electron beam with velocity $\approx 0.9995c$, c is the velocity of light, can result in a scattered beam of 3×10^{18} Hertz. Although, this proposal has been successful in producing the generation of gamma ray photons with good polarization and monochromaticity properties [79], it requires refinements due to the energy spread of the interacting electron beam, and some other effects. But it is hoped that the fundamental physics will not be changed by these effects.

4.2 Electron-Positron Graser

Some references, e.g., 69 and 70 , exploring the induced annihilation of particle-antiparticle bound states in positronium (Ps) to coherently amplify gamma radiation, propose a novel approach for developing the grasers. Moreover, refernces 42 and 71 propose a graser system through the relativistic electron-positron superpinch effect.

4.2.1 Induced Annihilation of Para-Positronium Atoms

The ground state of the Ps atom splits into singlet (Para-Ps) and triplet (Ortho-Ps) energy levels. After annihilation Para-Ps decays (decay time, T_0 , about 10^{-10} second) into two photons of equal energy in opposite directions while in Ortho-Ps (decay time about 10^{-7} second) into three photons of different energies.

For Para-Ps atom annihilations the total cross-section for the scattering of a photon on a stationary Ps atom may be expressed as [69]

$$\sigma(x) = (\pi e^2 / m^2) \cdot f(x)$$

where $x = \hbar\omega/mc^2$ depends on the incoming angular photon frequency, ω , the angular frequency of photon, m and e are the electronic mass and charge respectively, c is the velocity of light and $f(x)$ is a function in x . The function $\sigma(x)$ varies as

$$\sigma^-(x) \sim 1/x; x \rightarrow 0$$

$$\sigma^-(x) \sim 1/x^2; x \rightarrow \infty$$

The number of annihilations per second of Para-Ps atoms in a black-body radiation field can be expressed as

$$dN/dtdx = (-1/\pi) \cdot (mc/\hbar)^2 \alpha^6 \cdot (Nx f(x)/(\exp(ax) - 1))$$

where N is the total number of Para-Ps atoms, \hbar the Planck's constant, α the fine structure constant and

$$a = mc^2/kT_B$$

where k_B is the Boltzmann constant and T is the temperature of the medium.

For $a \gg 1$, i.e., $kT_B \ll mc^2$, or at low density, the number of annihilations per second can be given as

$$dN/dT = (-\pi/6) \cdot (\alpha^6/a^2) \cdot (mc/\hbar)^2 \cdot N$$

while in the limit $a \ll 1$, i.e., $kT_B \gg mc^2$, it is

$$dN/dT = (-\alpha^6/\pi) \cdot (mc/\hbar)^2 \cdot (\ln a/a) \cdot N$$

Thus, for $a \gg 1$, the spontaneous decay rate ($= -N/T_0$) dominates over the induced annihilation whereas the limit $kT_B \gg mc^2$ is important for developing a graser because here the induced decay dominates spontaneous decay. It has also been shown that the possibility for induced two photon decay of Para-Ps atoms as a graser is maximum at incident photon frequency of $\nu_c/2$, where ν_c is the Compton frequency for Para-Ps atoms [69]

At Lawrence Livermore National Laboratory scientists are also planning to start experimental work for developing the electron-positron graser [30].

It can also be shown [70] that the coherent amplification of gamma radiation is possible for a dense ($n > 10^{16} \text{ cm}^{-3}$) and cold ($T < 10^4 \text{ K}$) electron-positron plasma. Such conditions can be achieved in astrophysical objects such as pulsars, white dwarfs or black holes.

4.2.2 Relativistic Electron-Positron Graser

A proposal [42,71] for developing an electron-positron graser has been given on the basis of relativistic electron-positron superpinch leading to a population inversion into a dense plasma state. The superpinch is achieved through the fusion of two relativistic electron and positron beams of equal energy and density (see Fig. 2). Then under certain conditions, it will shrink in its diameter by relaxing heat through the emission of synchrotron radiation. In the relativistic case the annihilation cross-section is

$$\sigma_a = \frac{n_e (1 - v^2/c^2)^2}{m^2 c^2} \ln(2(1 - v^2/c^2))$$

where v is the relative velocity of the electron and positron beam, while in the non-relativistic case it is $(n_e / m c^2)$. Therefore, this reduced cross-section increases the time required for pair annihilation ($2/n \sigma c$; n is the number

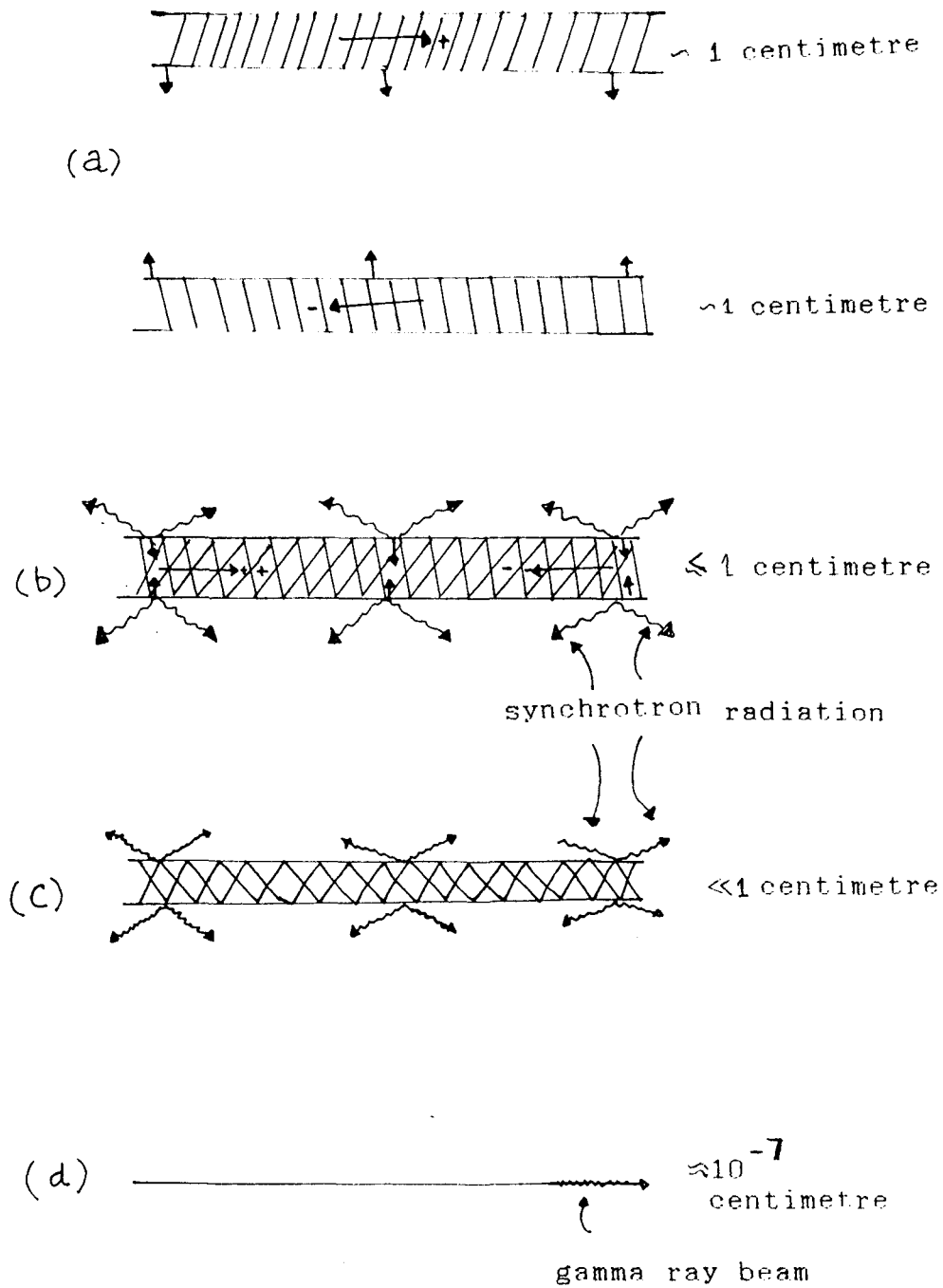


Fig. 2: Superpinch of two electron and positron beams

density of plasma) such that it becomes much larger than the time needed for the pinch of initial diameter, $2r$, to fuse (about $r mc(1 - v^2/c^2)^{-1/2} e$). And hence in such plasmas the very high densities without destroying the population inversion can be achieved. To explore this idea in a graser, the reduced axial motion of the electrons and positrons is required. For that it has been shown that for collapsing the two relativistic electron-positron beams of size 1 centimetre to 10^{-7} centimetre the electron-positron plasma would have to be established by the fusion of two 1.7×10^5 Amperes, 70 MeV electron-positron beams, which may be possible by exploring the very large magnetic fields of about 10^{12} Gauss available in plasmas.

4.3 Other Ideas

Exploiting the large magnetic fields generated in laser-produced plasmas, Loeb and Elizer^[72] show the possibility for constructing a plasma wiggler that may cause electron beams of 10^9 eV range to emit coherent gamma rays.

A proposal given by Vysotskii and Kuzmin^[73,74] shows the possibility of stimulated coherent gamma ray emission at wavelength about 1 nanometre by channeling a relativistic 50^8 MeV positron beams of current density about 10^8 Amperes per centimetre² through the zeolyte (asbestos fibre) medium.

C H A P T E R - 5

PROPOSED APPLICATIONS OF GRASERS

The high energy, short wavelength, coherent, monochromatic and directional graser beam may bring a new revolution in modern science and technology as the optical lasers did in the early 60's. Although, the full range of graser applications can only be predicted on the basis of its development, it is a human propensity to speculate the applications on the basis of present understanding [83].

A graser proposed on the basis of nuclear transitions can store about 3×10^{21} Watts per litre of its active medium, which is about 0.03% of the total electromagnetic power radiated by the Sun. This corresponds to an output energy of about 10^{12} Joules per pulse [27,84]. Such grasers would make a powerfull package which can be exploited in a space satellite to destroy enemy weapons [84].

Such high energetic grasers may destroy large asteroids approaching dangerously near the earth and thus might save humanity one day. Similarly, they may also be used in removing junk in near-earth and geo-synchronous orbits and in other macro-engineering projects.

Grasers may possibly also have some significant applications in space-ship propulsion. Lasers have already been proposed as a tool to propel an interstellar probe [85].

Grasers might explore some new types of startling phenomena in nuclear physics and may also be able to have an impact on nuclear spectroscopy [87] Balko has indicated the possibility of exploring the strong Mossbauer radiations from grasers as an efficient tool for classifying the various dynamic processes occurring in molecular systems

The high momentum of graser photons might exert very strong forces and hence may be exploited in isotope separation, novel vacuum pumps, cavitation and novel accelerators The graser photons may be able to provide escape or orbital velocity to dust particles and hence may be used by humanity in removing the effects of nuclear winter [83,86] which might be created after a major nuclear war

Graser beams may be exploited in producing large pressures which might be used for fusion of nuclei, metallic hydrogen and inverse beta-decay

Graser interferometry may be a powerful probe for the detection of gravitational waves.

The short wavelengths of graser beams may yield fruitful applications in determining crystal structures and in removing the crystal defects

Just like some proposals for developing a graser have explored the fusion processes, the graser may also be an efficient tool for plasma heating

Non-linear effects in gamma-ray region might be useful

in communication.

Grasers may be useful in molecular holography but it remains to be seen whether molecules can be saved from [1] destruction before the formation of the hologram .

CHAPTER - 6

ELECTRODYNAMICS IN A GRASER FIELD

6.1 Background to the Problem

The effect of a non relativistic charge particle on an electromagnetic plane wave is described in terms of the Thomson and Rayleigh scattering phenomena. But these scattering phenomena do not exhibit the effect of the electromagnetic wave on the charge. Here in subsection 6.1.1 we wish to build a sufficient background to the problem.

Electrodynamics in a Graser Field, through first discussing the behaviour of the charge particle in the electromagnetic plane wave. In subsection 6.1.2 is presented a treatment of the same problem in a laser beam.

6.1.1 Charge Particle Dynamics in the Electromagnetic Field of a Plane Wave [88]

A general relativistic equation of motion for the particle in an electromagnetic field is

$$dp/dt = d[mv/(1 - v^2/c^2)^{1/2}]/dt = q(E + v \times B) \quad (1)$$

where p is the momentum of the particle of rest mass m and charge q and moving with velocity v in electric field intensity E and magnetic field intensity B at time t . c is

the velocity of light.

For a plane wave propagating along \hat{k} , the magnetic and electric field vectors are related as

$$\vec{B} = \hat{k} \times \vec{E}/c \quad \dots(2)$$

Substitution in equation (1) gives

$$d[\vec{v}/(1 - \vec{v} \cdot \hat{k}/c)^{1/2}]/dt = (q/m) (1 - \vec{v} \cdot \hat{k}/c) \vec{E} + (q/mc) (\vec{v} \cdot \vec{E}) \hat{k} \quad \dots(3)$$

or,

$$d[\hat{k} \cdot \vec{v}/(1 - \vec{v} \cdot \hat{k}/c)^{1/2}]/dt = (q/mc) (\vec{v} \cdot \vec{E}) \quad \dots(4)$$

The energy equation

$$d[mc^2/(1 - \vec{v} \cdot \hat{k}/c)^{1/2}]/dt = q\vec{v} \cdot \vec{E}$$

gives

$$K = [1 - \hat{k} \cdot \vec{v}/c]/(1 - \vec{v}^2/c^2)^{1/2}$$

as a constant of motion.

The velocity of the particle may be expressed in terms of the plane wave as

$$\begin{aligned} \vec{v} &= d\vec{r}/dt = (d\vec{r}/dl) \cdot (dl/dt) \\ &= \omega (1 - \vec{k} \cdot \vec{v}/c) d\vec{r}/dl \end{aligned}$$

where $l = \omega(t - \hat{k} \cdot \vec{r}/c)$ is the phase factor.

Thus equation (3) results in a differential equation for determining \vec{r} as a function of phase parameter l .

$$d^2\vec{r}/dl^2 = (q/Km\omega') [\vec{E} + (\omega/c) (\vec{E} \cdot d\vec{r}/dl) \hat{k}] \quad \dots(5)$$

For a linearly polarized electromagnetic wave propagating along z direction the electric field intensity varies as

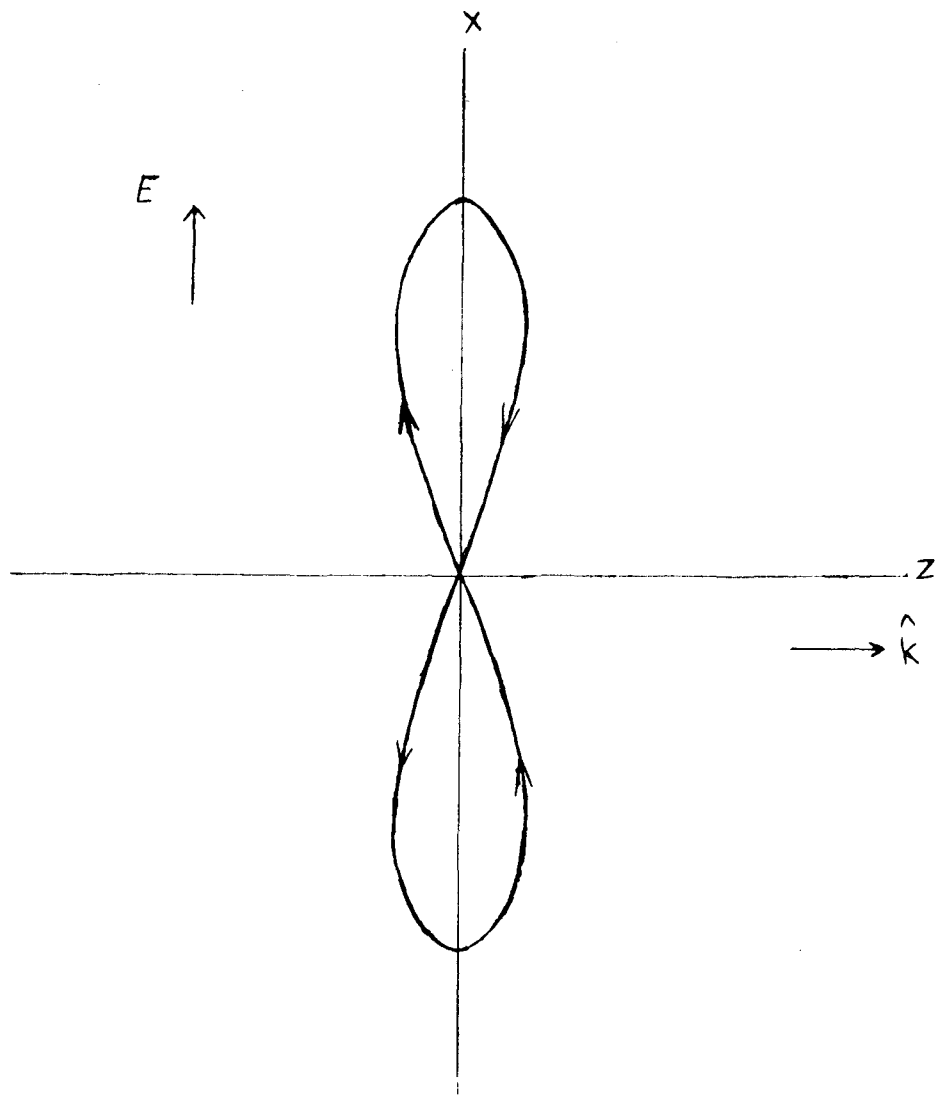


Fig. 1: Behaviour of electron in the electromagnetic field of a plane wave

$$\vec{E} = E_0 \cos \omega(t - z/c)$$

where E_0 is the constant amplitude of the wave.

Substitution in equation (5) yields the components as

$$\begin{aligned} d^2x/dl^2 &= (qE_0/Km\omega^2) \cos \omega(t - z/c) \\ d^2y/dl^2 &= 0 \end{aligned} \quad \dots(6)$$

$$d^2z/dl^2 = (qE_0/cKm\omega) \cdot (dx/dl) \cos \omega(t - z/c)$$

or,

$$\begin{aligned} x &= (-qE_0/Km\omega^2) \cos \omega(t - z/c) \\ y &= 0 \end{aligned} \quad \dots(7).$$

$$z = (-q^2 E_0^2 / 8K^2 c m \omega^3) \sin 2\omega(t - z/c)$$

Hence, the motion of charge particle in the electromagnetic field of a plane wave is an oscillating trajectory in xz plane as shown in Fig. 1.

6.1.2 Charge Particles in a Laser Field

The object of this subsection is to discuss the behaviour of a charge particle, specifically an electron, in the electromagnetic field of the laser. According to the dipole approximation, requiring the radiation field wavelength to be larger than the electron mean free path, the laser beam can be represented by a classical plane electromagnetic wave.

linearly polarized plane wave is

$$(i\hbar\partial/\partial t - \bar{H})\psi(r,t) = 0 \quad \dots(1)$$

where $\psi(r,t)$ is the wave function corresponding to the Hamiltonian operator \bar{H} for the electron interacting with the wave, given by

$$\bar{H} = (1/2m)[\bar{P} + (q/c)\bar{A}(r,t)]^2 \quad \dots(2)$$

where \bar{P} is the momentum operator and $\bar{A}(r,t)$ is the potential applied by the laser beam expressed as

$$\bar{A}(r,t) = (c/\omega) \bar{E}(r) \cos(\omega t - \bar{k}\bar{r}) \quad \dots(3)$$

\bar{k} is the propagation vector of the wave of frequency ω , $\bar{E}(r)$ is the spatial dependent electric field.

Thus equation (1) becomes

$$\{i\hbar\partial/\partial t - [(P^2/2m) + (q^2 E^2(r)/4m\omega^2) + (q/m\omega) \cos(\omega t - \bar{k}\bar{r}) \bar{E}(r), \bar{P}^2 + (q^2 E^2(r)/4m\omega^2) \cos 2(\omega t - \bar{k}\bar{r})]\} \psi(r,t) = 0 \quad \dots(4)$$

In equation (4) the last two terms represent the oscillations in the electron motion and are short enough to be neglected in comparison with the first two terms appearing within the square bracket. However, in case of large amplitude laser waves these oscillating terms might become comparable to others. Therefore, for small amplitude waves the term $(q^2 E^2(r)/4m\omega^2)$ is the only potential for the electron motion in the laser field.

Now, if $\hbar p$ is the electron momentum governed by

the hamiltonian

$$H_C = \frac{P^2}{2m} + (q E(r)/4m\omega)^2 \dots (5)$$

The reference 89 shows its effect which alters the momentum of an electron entering into the laser field.

[90]

In 1986, Freeman et al. have demonstrated an experiment for the large angle scattering of free electrons from a focused laser beam. They have found that at all low laser intensities the electrons possess their initial velocities but become parallel to the laser's polarization vector while at higher intensities (about 10^{13} Watts per centimetre²) the collimation of electrons along the polarization vector goes off. They have further studied this effect as a function of laser intensity and found the angular distribution function of scattered electrons as a composite function of initial electron momentum and the light intensity.

6.2 Interaction of an Electron with a Graser Field

6.2.1 General Description

In the preceding section we have been talking about the interaction of a charge particle with an electromagnetic wave and also about the response of free electrons to a laser beam. Here we wish to extend the problem to interaction with a graser field. As a part of this work we have attempted a few model calculations for the behaviour of a non-relativistic charge particle in the electromagnetic field of a graser. We are assuming the radiation field to be a plane wave and the free electron to be a particle.

The expression for the time and spatially dependent electric field in a plane wave travelling in the positive z direction and polarized along the x direction is given by

$$E(x,t) = E_0 \cos(kz - \omega t - \phi) \exp(-x^2/R^2) \quad \dots(1)$$

where E_0 is the constant electric field of the wave, k is the magnitude of the propagation vector (\vec{k}) of the wave, ω and ϕ are its angular frequency and phase parameter, respectively, and R is a parameter representing the beam-width.

The magnetic field strength of the wave can be represented (in SI units) as

$$B = E/c \quad \dots(2)$$

and the direction of magnetic field vector will always be

perpendicular to the direction of electric field vector and to the propagation vector.

If the non-relativistic electron is moving in the x direction then the equation of motion of the electron inside the beam is governed by the Lorentz force equation

$$F = q(E + vB) \quad \dots(3)$$

where v is the velocity of the electron interacting with the graser beam.

But the study of radiations ejected from the moving charge particles in the field of an electromagnetic wave yields an additional force acting on the particle which alters its motion, commonly known as the radiation reaction force (F_{rad}) [91]. So the total force exerted on an electron in the graser field should be

$$F_{\text{total}} = F + F_{\text{rad}}$$

$$F_{\text{total}} = q(E + vB) + (2q^2/3c) \cdot d^2v/dt^2 \quad \dots(4)$$

The force exerted by the magnetic part of the graser field

$$F_{\text{mag}} = qE(v/c) \quad \dots(5),$$

because of the factor v/c being very small compared to unity for a non-relativistic particle, can be neglected in comparison with the force of the electric part of the graser field.

And for graser parameters it can be shown that the radiation reaction part of the total force being exerted on

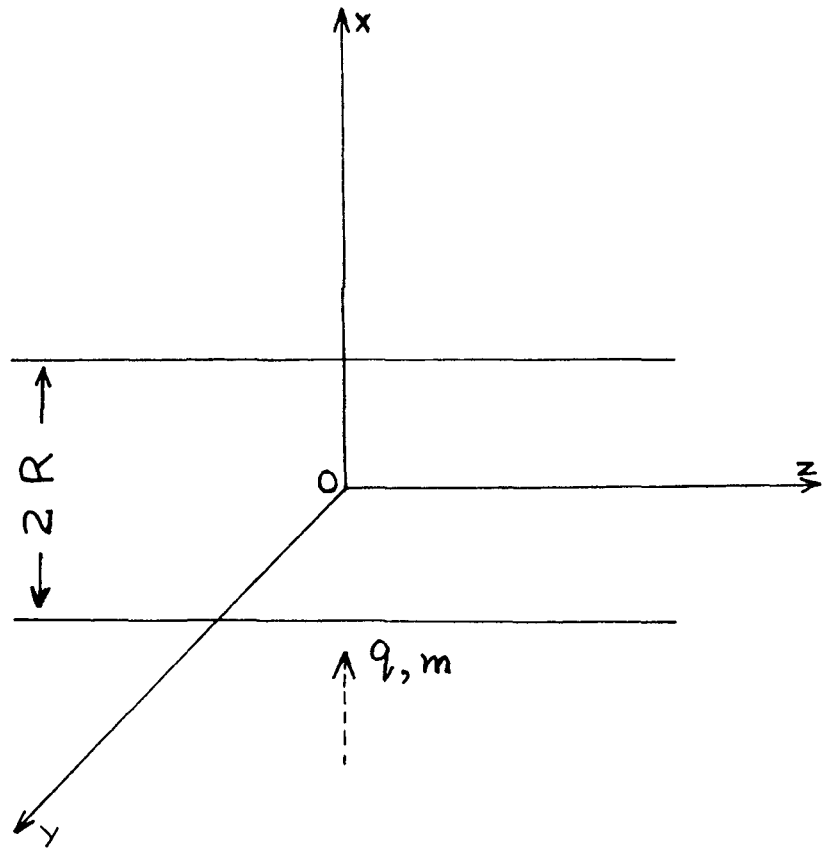


Fig. 2: A graser beam and the incoming particle

the moving electron is much smaller than even the magnetic force (see Appendix A). Therefore, the electric force acting on the electron in a graser field is the most important force for studying its behaviour.

Thus on the basis of the discussion made so far we are left with

$$d(mv)/dt = qE$$

or,

$$\begin{aligned} d(mv)/dt &= qE(x,t) \\ &= qE_0 \cos(kz - \omega t - \phi) \exp(-x^2/R^2) \quad \dots(6). \end{aligned}$$

Hence, the final equation of motion for the electron interacting with the graser beam is

$$d^2x/dt^2 = (qE_0/m) \cos(kz - \omega t - \phi) \exp(-x^2/R^2)$$

Now, if one sees the interaction at the point $z = 0$ (see Fig. 2) then equation of motion can be rewritten as

$$d^2x/dt^2 = (qE_0/m) \cos(\omega t + \phi) \exp(-x^2/R^2) \quad \dots(7).$$

6.2.2 Calculation and Results

In order to study the response of a free electron in the field of a graser in non-relativistic regime one has to solve the differential equation (7). Since, such a type of equation can not be solved directly, one has to solve it

numerically.

First, we will attempt to solve it with a model approximation, i.e., by ignoring the spatial dependence of the oscillating electric field so the equation (7) reduces into

$$\frac{d^2 x}{dt^2} = \left(\frac{qE_0}{m\omega} \right) \cos(\omega t + \phi) \quad \dots(8)$$

yielding the velocity of electron $v(t)$ inside the graser field as

$$V(t) = \frac{dx}{dt} = \left(\frac{qE_0}{m\omega} \right) \sin(\omega t + \phi) - \left(\frac{qE_0}{m\omega} \right) \sin\phi + v_0 \quad \dots(9)$$

where v_0 is the initial electron velocity at time $t = 0$.

The position $x(t)$ is

$$x(t) = \left(- \frac{qE_0}{m\omega^2} \right) \cos(\omega t + \phi) - \left(\frac{qE_0}{m\omega^2} \right) \sin\phi + v_0 t + \left(\frac{qE_0}{m\omega^2} \right) \cos\phi \quad \dots(10).$$

Numerically for graser parameters, the photon energy is in the range of about 10 keV to about 10 MeV. The angular frequency, ω , corresponding to the about 40 keV per quantum is 2×10^{19} radians per second. And as a typical case the beam-width parameter of the graser beam, R , can be taken as 5.0×10^{-4} metre. Further, in Chapter 5 it has been given that the output power of a graser beam may be as great as 3×10^{21} Watts per litre of the graser material [27,84] which would correspond to 1.7×10^{15} Volts per metre of the electric field, E .

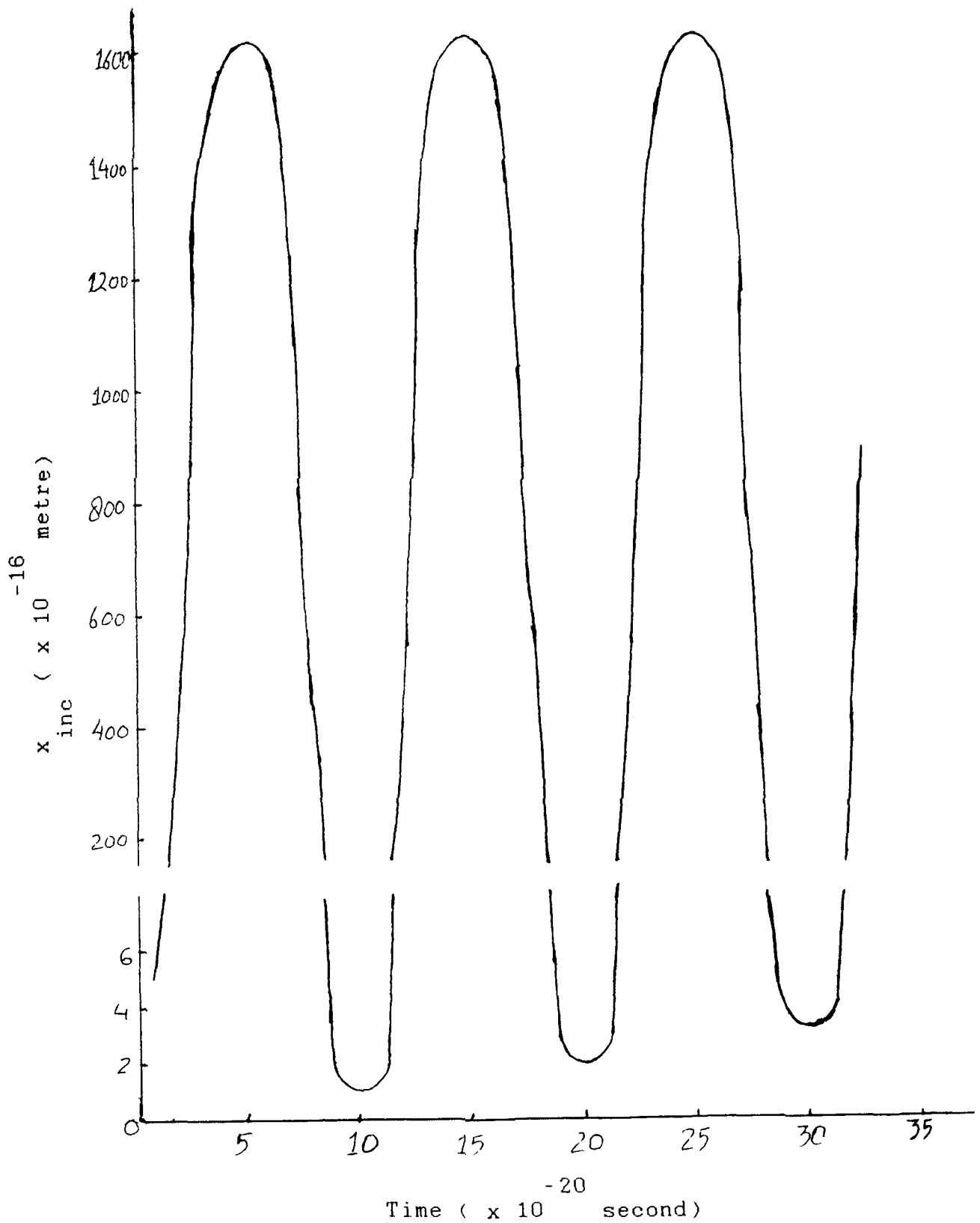


Fig. 3: The oscillatory motion of an electron inside the spatially independent graser beam (phase = 0 radians) field. The ordinate x_{inc} is the increment in the position of the electron in the graser field, i.e., $x(t) = x + x_{inc}$.

In the non-spatially dependent electric field case there is no field outside the graser beam-width parameter, R . So in order to see the behaviour of the electron in the field the initial value of the position and velocity of electron can be taken as

$$x(t = 0) = x_0 = -R$$

$$= -5.0 \times 10^{-4} \text{ metre}$$

and, $v(t = 0) = v_0 = 10^3$ metres per second ... (11).

Let T , the transit time, be the time taken by the electron in crossing the graser field (beam) from $x = -R$ to $x = R$, which can be evaluated as

$$T = \frac{2.R}{v_0 - (qE_0/m\omega) \sin \phi} \quad \dots (12).$$

This will be true provided

$$(qE_0/m\omega^2) \ll R,$$

which is well fitted in our calculation.

Equation (9) gives the velocity at time T as

$$v(T) = (qE_0/m\omega) \sin(\omega T + \phi) - (qE_0/m\omega) \sin \phi + v_0 \quad \dots (13).$$

Equations (13) and (9) for ϕ , the phase parameter, $= 0$ yield

$$v(T)/v_0 = (qE_0/m\omega v_0) \sin \omega T + 1 \quad \dots (14)$$

or from the maxima-minima principle.

$$(v(T)/v_0)_{\text{Maximum}} = 1$$

and,
$$T = \frac{2R}{V_0} \dots (15).$$

A positive value of the transit time, T , represents the transmission of the particle through the graser beam, while for $\phi = \pi/2$ equation (12) gives reflection of the electron by the field of the graser beam.

Fig. (3), representing the motion of the electron, shows the oscillatory motion inside the beam having zero phase, ϕ , and for 10^3 metres per second initial velocity. It shows a microscopic view of the motion for a short time scale, i.e., 2.5×10^{-20} second. And the transit time can be calculated directly from equation (15) as about 10^{-7} second. Since in this case the electric field strength is independent of spatial parameters, this oscillatory motion of the electron inside the beam is not going to alter even at the time of leaving the beam.

From equation (12) it is clear that if the phase parameter of the graser beam is changed to $\pi/2$ radians, transmission of the electron into the beam can not occur unless the initial electron velocity exceeds the value $(qE/m\omega)$. For graser parameters it would be about 4.76×10^6 metres per second. Fig. (4) shows the behaviour of the electron inside the beam at such higher initial velocities, viz., for $v_0 = 5.0 \times 10^6$ metres per second. However at this initial velocity the magnetic field and relativistic

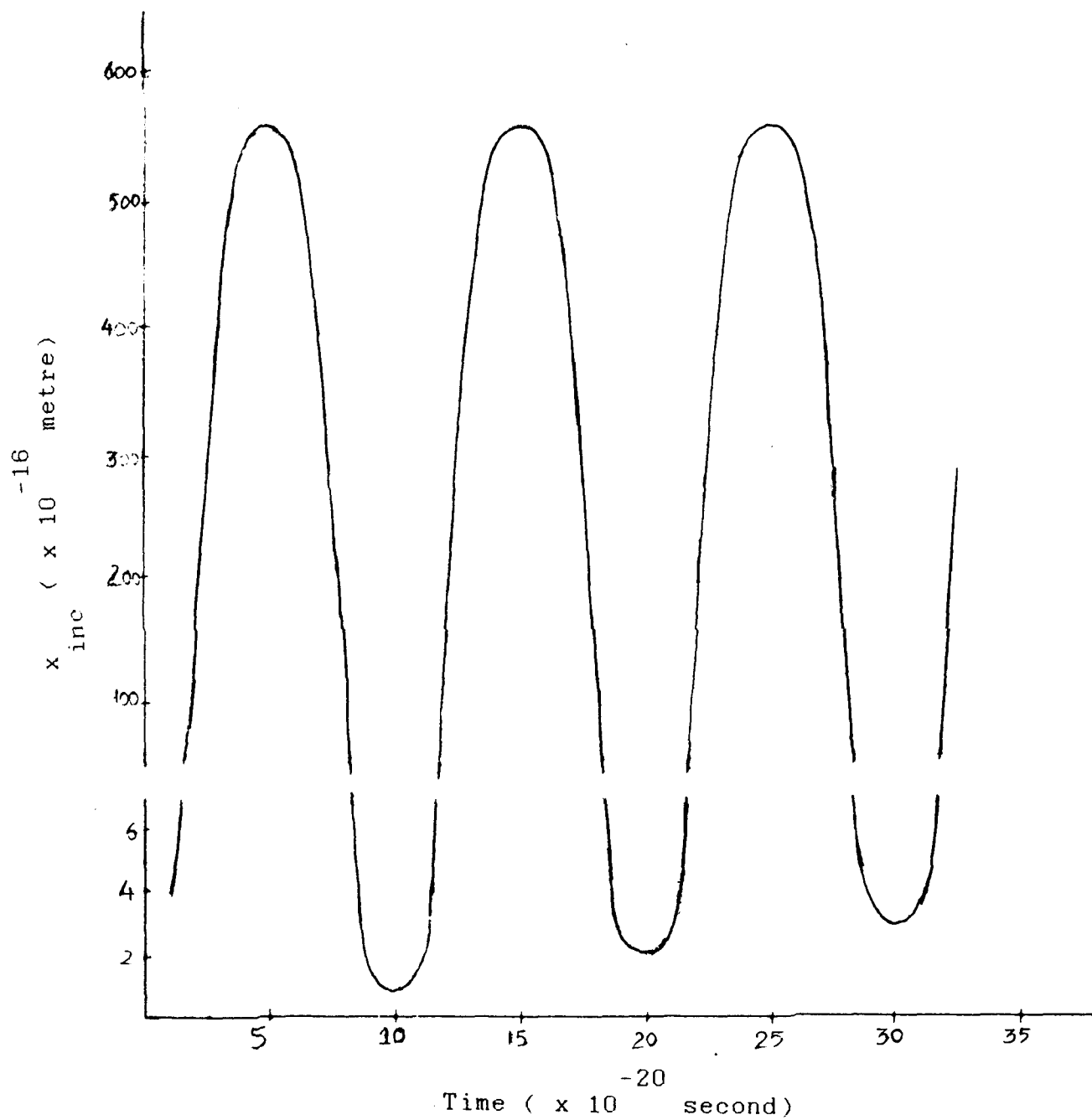


Fig. 5: The oscillatory motion of an electron inside the graser beam (phase = 0) field.

corrections to the given calculation become about 1.7% of the electric field force, which is again short enough to be neglected.

After discussing the simplest model calculation for the response of an electron into the graser beam field in which the spatial dependence of the electric field strength has not been taken into account, we would like to go back to equation (7) for further calculations by considering the spatial dependence of the electric field strength.

(I) Zero Phase ($\phi = 0$ radians)

Equation (7) can not be solved directly unless some numerical techniques are involved. We have attempted to solve it with the fourth-order Runge-Kutta method [92,93] using the Digital Equipment Corporation's VAX VMS-11/780 version 2.0 computer system. Fig. (5), analogous to the Fig. (4), represents the microscopic view for short time scale for the behaviour of an electron inside the graser field in which the spatial dependence of the electric field strength has been considered. It shows the motion as an oscillatory one similar to the case where the electric field strength was taken to be spatially independent. And the only difference is exhibited in the amplitude of oscillations. Moreover, in this case the field is not only confined within the beam-width parameter but there will also

be a quite effective field strength outside the beam and can not be omitted.

Since, the electric field strength is not uniform in space, the amplitude of oscillations of the electron motion is not the same throughout the field. It first increases as the field increases and goes to maximum at the center and then again decreases as the field becomes small.

The above calculation has been checked by assuming a much larger beam-width parameter, i.e., $R' = 10^3 \times R$. This enables one to assume the electric field strength to be uniform over a short distance (10^{-3} metre) around the centre of the beam. In this case we found the same results as for the motion of electron in the spatially independent electric field.

(II) $\pi/2$ Phase ($\phi = \pi/2$ radians)

Let us recall the case of spatially independent electric field strength and $\pi/2$ radians graser beam phase. We have seen in this case that the electron can not enter into the graser field until its initial velocity exceeds $(qE_0/m\omega)$. Similarly, for spatially dependent electric field strengths of the graser field one must have a corresponding cut-off value for the initial electron velocity. In this case we find this value to be about 1.76×10^6 metres per second, below which the electron is reflected from the oscillating graser field.

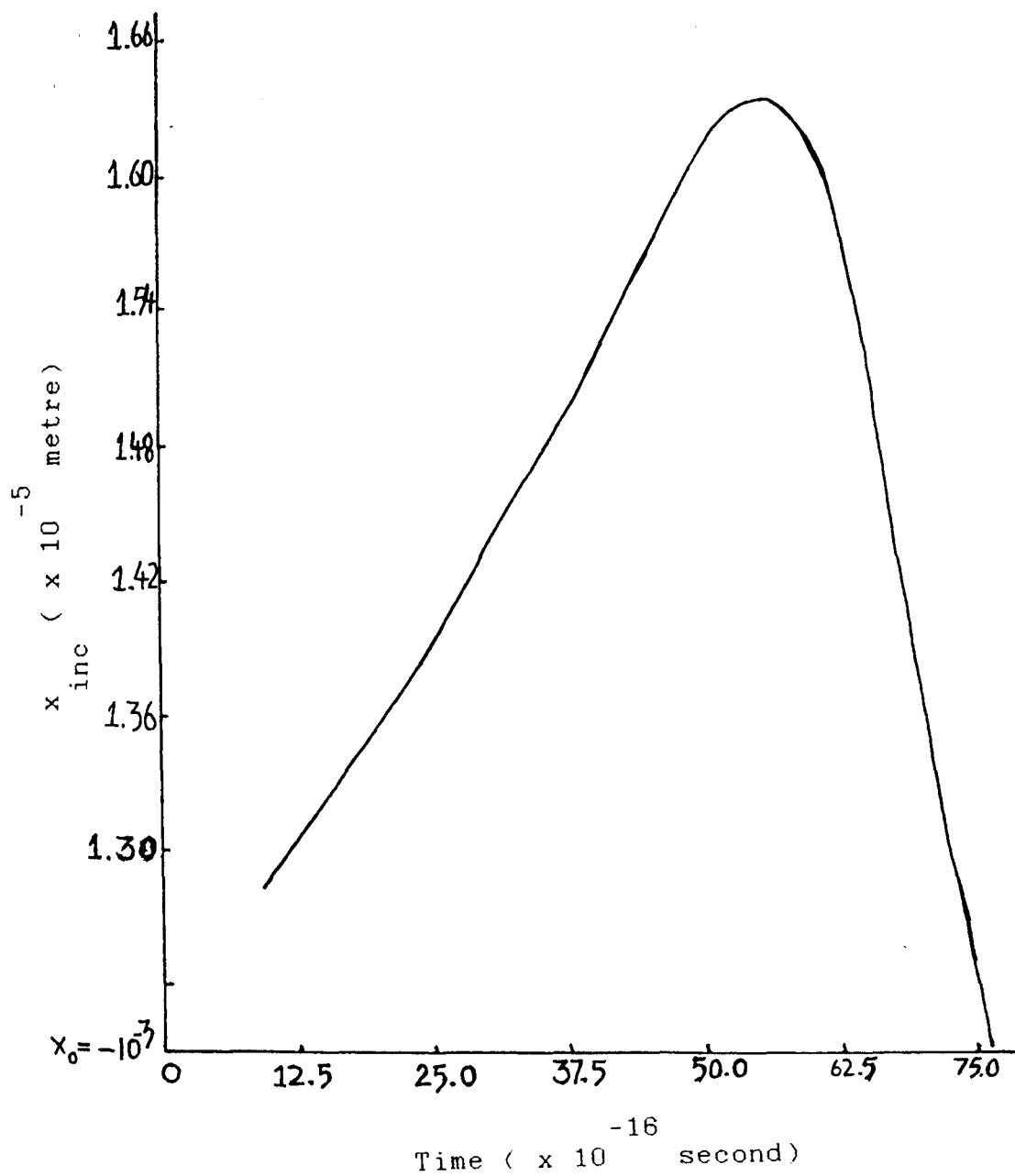


Fig. 6: The reflectory motion of an electron in the graser beam (phase = $\pi/2$ radians) field.

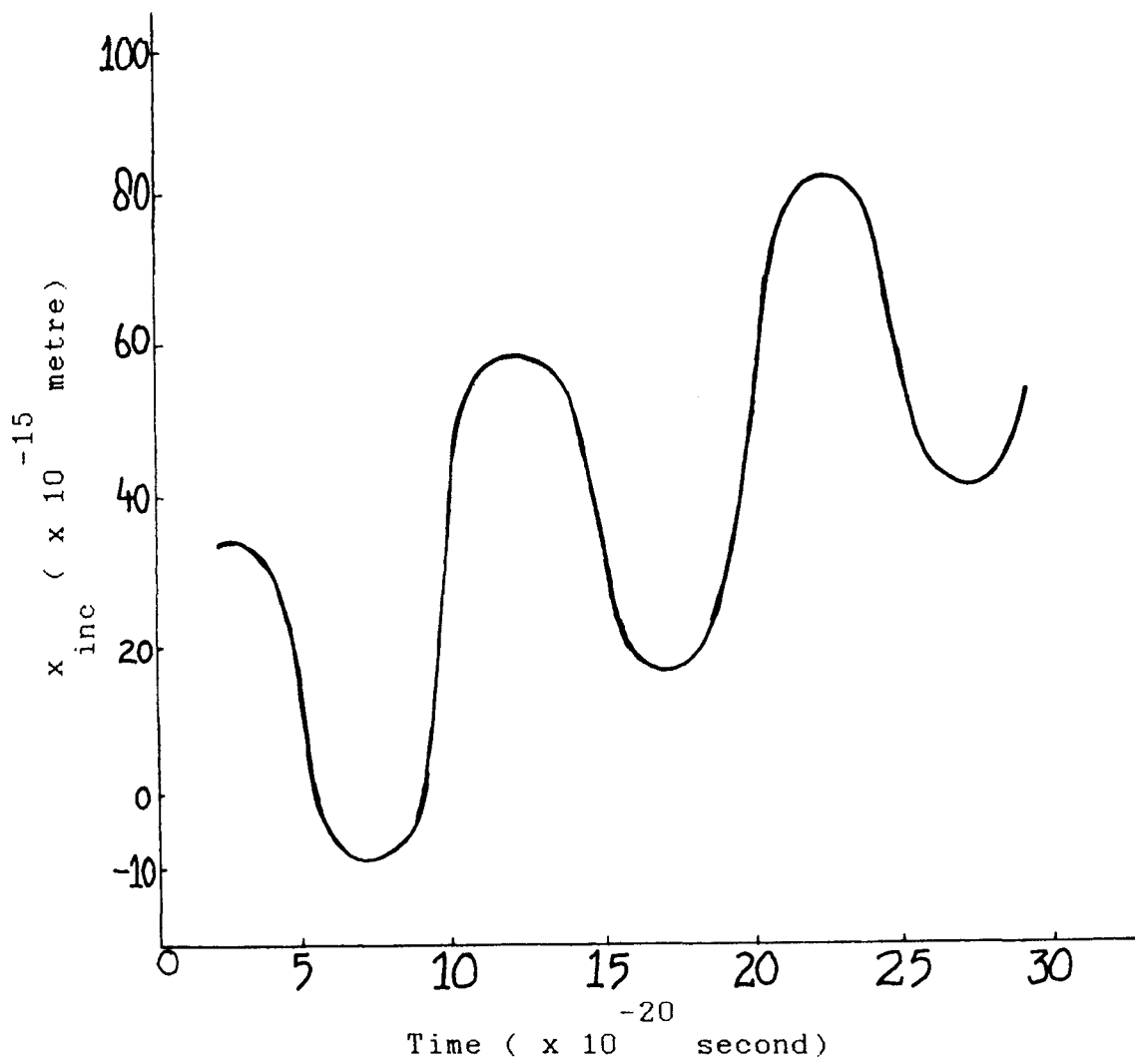


Fig. 7: The oscillatory motion of an electron in the graser beam (phase = $\pi/2$ radians).

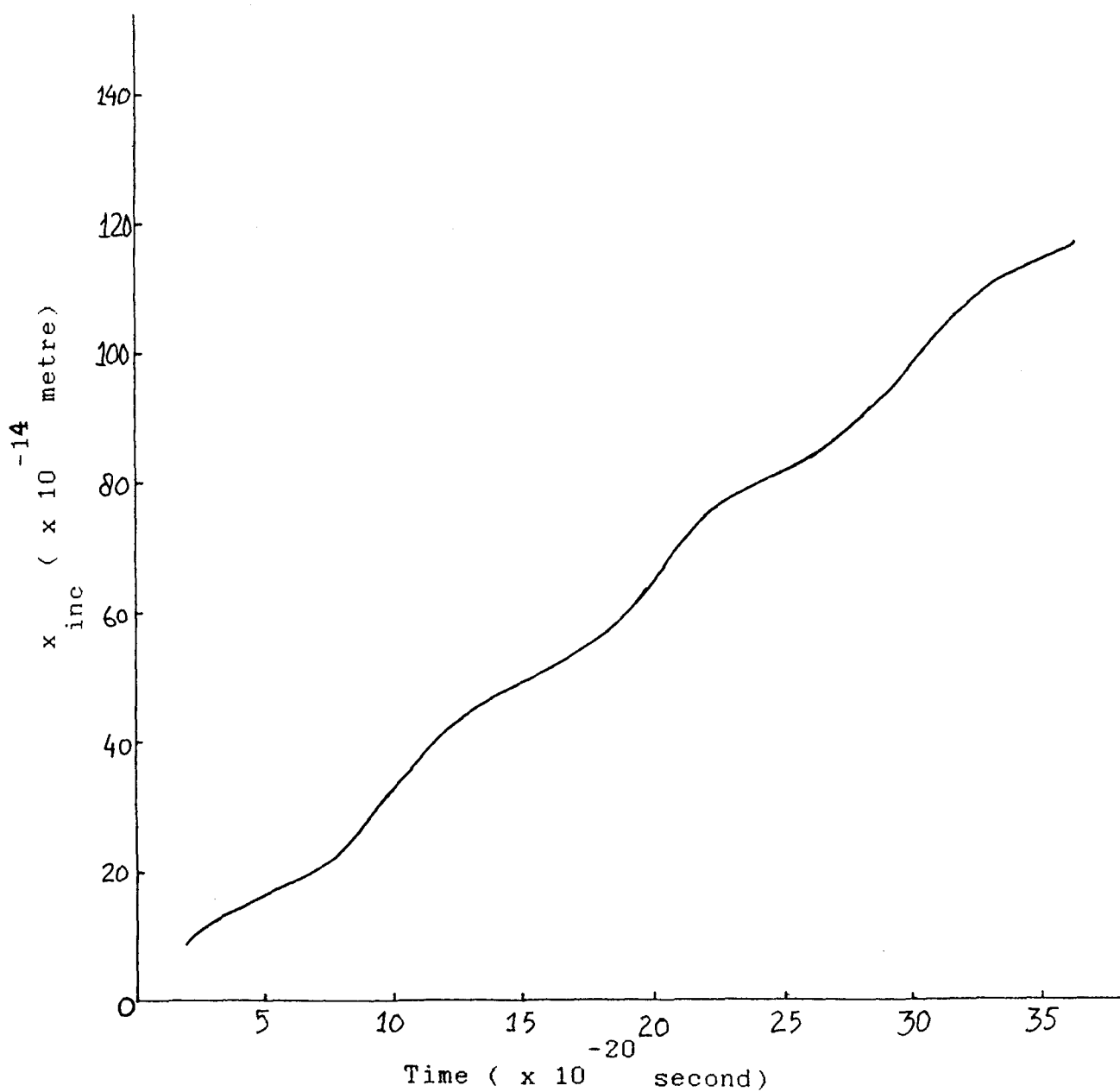


Fig. 8: The translational motion of electron inside an electron inside the graser beam (phase = $\pi/2$ radians).

Fig. 6 shows the interaction of the electron with the graser field at 10^5 metres per second initial velocity, v_0 when the initial position of electron is $x(t = 0) = -10^{-3}$ metre.

From this figure it is apparent that the electron first enters the field with its own linear momentum but as soon as the graser field overcomes it, the electron is reflected in the backward direction. It is found that the reflectory nature of the electron into the graser field is exhibited until v_0 is raised upto 1.7556×10^6 metres per second. At this cut-off initial velocity the oscillatory motion of the electron inside the graser field is exhibited (see Fig. 7). Fig. 8 shows the translational motion of the electron inside the graser field at much higher initial velocity, i.e., at 5×10^6 metres per second.

Fig. 9 shows the different transit times, T , in which the electron crosses the graser beam (10^{-3} metre). For that part of the calculation we had supposed the initial electron position to be $x_0 = -5 \times 10^{-4}$ metre (in order to reduce the large CPU time required by the computer). We see that for higher initial velocities (above 3×10^6 metres per second) the transit time is approximately same as given by the simple kinematics, $T = 2R/v_0$, but for little lower initial velocities the motion involves dynamics and the transit time abruptly increases as the initial velocity decreases. It

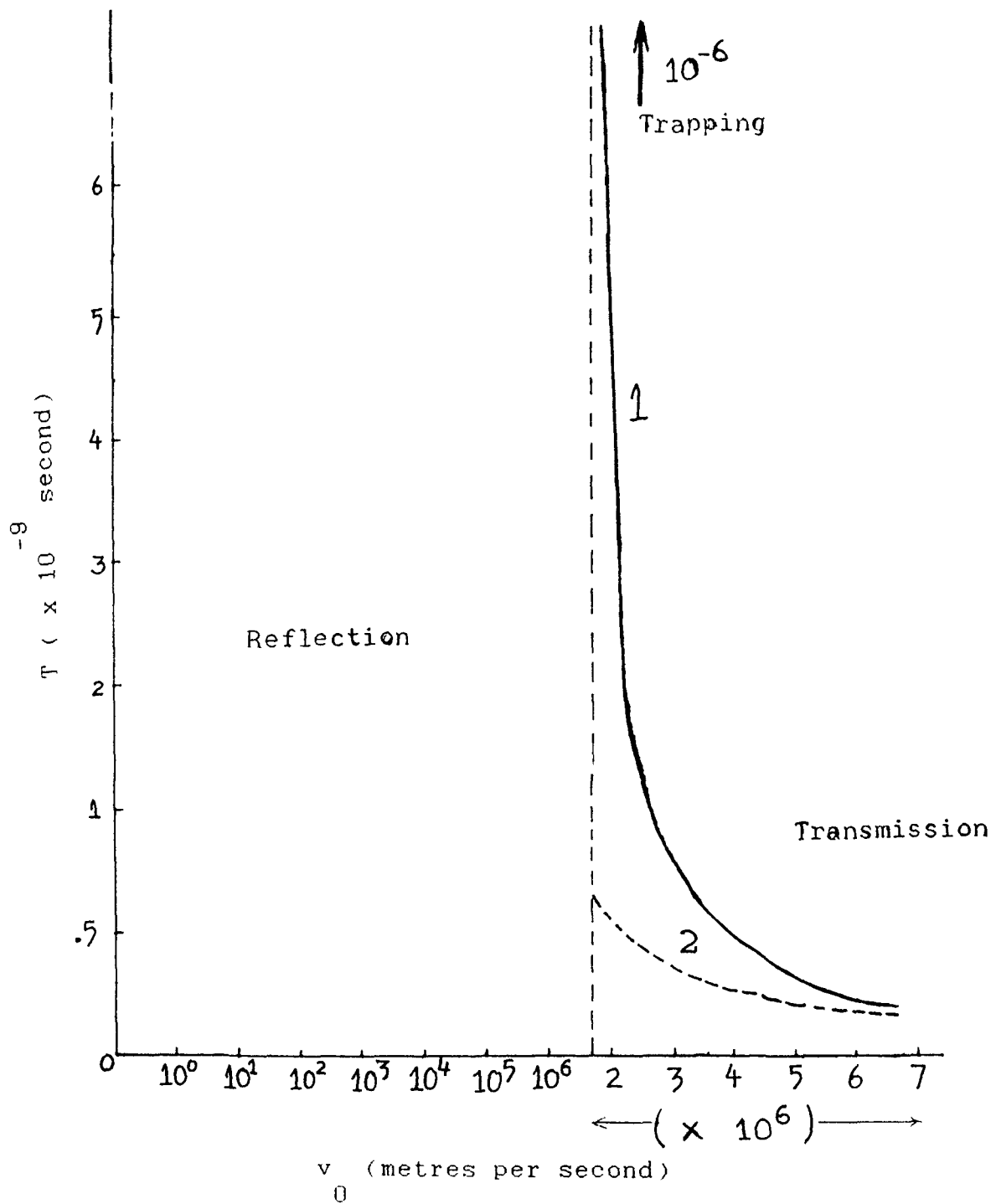


Fig. 9: The transit time (T) vs. initial velocity. Curve 1 is for the dynamical motion of the particle in the graser beam. Curve 2 is for purely kinematical motion ($T = 2R/v_0$)

also gives an idea of the trapping of an electron for a long time (at least for 10^{-6} second) inside the graser beam at initial velocities $> 1.7556 \times 10^6$ metres per second.

On assuming the large beam-width parameter ($R' = 10^3 \times R$) these results lead to the same outcome as in the case when the electric field is taken to be spatially independent.

6.2.3 Conclusion

In conclusion we would like to impress the reader's attention to the nonzero phase ($\phi = \pi/2$ radians) in which the reflection, transmission and the trapping of electron is exhibited as a function of the initial velocity of the incoming electron. Such type of phenomena are not observable in the case of ordinary lasers because their electric field strengths (about 10^4 Volts per metre) are not as high as the grasers (1.7×10^{15} Volts per metre).

In this model calculation we have assumed the interaction with the non-relativistic electron. But if the electron is taken to be relativistic then the relativistic correction to the Newton's law, magnetic field and radiation reaction forces, being exerted on the electron in graser field are to be considered. The inclusion of all these terms to the equation of motion, equation (7) will complete the classical treatment to the problem. Further, it will be an interesting idea to see the problem quantum mechanically [89].

APPENDIX - A

Comparison between Magnetic, Relativistic Correction and Radiative Reaction Forces Acting on the Electron in Graser Field in Addition to Equation (6) of Section 6.2

In equation (6) of section 6.2 we have taken the equation of motion for the non-relativistic electron in a graser field under the influence of electric force only. But there are some other forces acting on the electron in the field. And these forces may be appreciable to be considered in describing the behaviour of a relativistic electron. Here we wish to work out a priority-wise list of such forces, viz., magnetic force, relativistic correction to the Newton's law and the radiative reaction force, to be included in equation of motion.

(i) The Radiation Reaction Force (F_{rad})

[91] It is expressed as

$$F_{\text{rad}} = \frac{2}{3} \frac{q^2}{c} \cdot \frac{d^2 v}{dt^2} \quad \dots(16).$$

An electron interacting with the oscillating field also starts oscillations with the frequency of field. Therefore, the velocity of electron inside the graser field can be taken as

$$v \approx v_0 \cos \omega t \quad \dots(17)$$

where v_0 is the velocity of incoming electron. Thus F_{rad}

becomes

$$F_{\text{rad}} \sim 2q^2 \omega^2 v^3 / 3c^3 \quad \dots(18).$$

Thus for an electron with $v = 5 \times 10^6$ metres per second, it comes to be about 10^{-17} Newtons. While for a relativistic electron ($v = 0.5c$) it would be about 3×10^{-16} Newtons.

(ii) Relativistic Correction

According to Newton's law of motion the total force experienced by the relativistic electron in a field is

$$\begin{aligned} F_{\text{total}} &= dp/dt \\ &= d[mv (1 - v^2/c^2)^{-1/2}] / dt \\ &= d/[mv (1 + v^2/2c^2 - (1/8).(v^4/c^4) + \dots)] / dt \end{aligned}$$

where p is the electron momentum. Since v/c is shorter than unity so the higher power terms of $(v/c)^2$ appearing in the above series can be neglected and hence

$$F_{\text{total}} = dm v / dt + d(mv^3 / 2c^2) \quad \dots(19).$$

In this relativistic motion the first term represents the non-relativistic form of the equation of motion while the other term represents a relativistic correction to it. This correction can be expressed, using the equation (17), as

$$F_{\text{rel}} \sim (3m\omega^2 v^3 / 2c^2) \quad \dots(20).$$

Thus for a non-relativistic electron this correction is about

10^{-8} Newton. While for relativistic electron velocities it becomes comparable to the force exerted by the electric field, qE , which is of the order of 10^{-6} Newton.

(iii) The comparison between the magnetic force and relativistic correction can be made as

$$\begin{aligned} \frac{F_{\text{mag}}}{F_{\text{rel}}} &= (qvE/c) / (3m\omega v^3/2c^2) \\ &= (2qEc/3m\omega v^2) \end{aligned} \quad \dots(21)$$

The numerical substitution of terms appearing in the right hand side of this above equation at $v = 5 \times 10^6$ metres per second gives its value to be about 38, while at $v = 0.5c$ it is about 3.2×10^{-2} .

Thus for non-relativistic electron the magnetic field correction is more important than the relativistic correction while for relativistic electron velocities the relativistic correction to the equation of motion is more important.

(iv) A comparison between magnetic field and radiation reaction forces yields

$$\frac{F_{\text{mag}}}{F_{\text{rad}}} = 3Ec^2/2q\omega$$

to be about 10^{12} .

Thus on the basis of above discussion one can make a comparison between these three correction forces to the

equation of motion describing the behaviour of electron in the graser field. It is obvious that for a non-relativistic electron all these extra forces can be neglected in comparison to the electric force. But for a relativistic electron these extra forces become comparable to the electric force. Rankwise, the relativistic correction to the equation of motion stands first, the magnetic field correction lies on the second position while the radiative reaction force places last.

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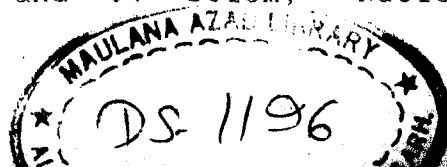
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